ELECTRON TRANSPORT COEFFICIENTS IN A NONEQUILIBRIUM WEAKLY IONIZED PLASMA IN ELECTRIC AND MAGNETIC FIELDS

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UDC 537.523.74

1. Introduction

A weakly ionized plasma in electric and magnetic fields, in which the electron energy distribution is nonequilibrium, is considered in the present article. Such a plasma is encountered in the ionosphere [1], MHD generators [2], and semiconductor devices. A nonequilibrium gas-discharge plasma in crossed electric and magnetic fields has recently evoked increased interest in connection with a number of technical applications [3, 4].

The phenomenon of anisotropy of electron diffusion in gases under the action of an electric field, which has found a theoretical explanation in [6, 7], was discovered experimentally in [5]. In [8] it was shown that the system of equations of electron transport in a weakly ionized, weakly nonuniform plasma in an electric field is reduced to one modified equation of continuity, which was obtained in the final form (without a magnetic field) in [9]. This is connected with the fact that the average velocity and the average energy of the electrons are uniquely determined by the external electric field and the cross sections of electron scattering on atoms and molecules. According to [9], not only nonuniformity of the electron density but also nonuniformity and nonsteadiness of the parameter E/N (E is the electric field strength and N is the density of neutral particles) result in a renormalization of the electron flux along the electric field, i.e., the electron "thermodiffusion" in the electric field is also anisotropic. A large number of articles [7, 9] have been devoted to the calculation of electron transport coefficients in an electric field (without a magnetic field).

A modified equation of electron transport in a weakly ionized plasma in electric and magnetic fields was derived in [10], equations were given for determining the electron transport coefficients, and the stability of a weakly ionized plasma was investigated on the basis of the new transport equation.

In the present article we consider the limit of a weak magnetic field, when the electron cyclotron frequency is large compared with the frequency of electron momentum transfer. Explicit expressions are obtained for the electron transport coefficients in the model case when the collision integral has a divergent form while the cross sections of electron scattering on neutral particles are power-law functions of the electron velocity. A collision integral of this kind occurs if the main losses of electron energy are connected with elastic collisions (atomic gases). In molecular gases the collision integral is reduced to the same form under certain conditions with the excitation of rotations and vibrations. Only the transverse diffusion coefficient was calculated earlier [7] from a set of electron transport coefficients for a strong magnetic field.

2. Fundamental Equations

We obtain the equations for determining the electron transport coefficients in a nonequilibrium weakly ionized plasma in electric and magnetic fields following [10].

If the electrons acquire an energy considerably higher than the thermal energy ($eE\lambda_u \gg T$, where λ_u is the relaxation length of the average electron energy, e is the electron charge, and T is the gas temperature) over a length λ_u under the action of the electric field, then the average electron energy considerably exceeds the energy of the heavy particles [11]. For a sufficiently low degree of ionization α_i ($\alpha_i \lessdot 10^{-6}$–$10^{-4}$ for various gases) the electron mean free path with respect to electron–electron scattering exceeds λ_u, and the electron

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 20-24, May-June, 1984. Original article submitted May 24, 1983.

 0021-8944/84/2503-0346$08.50

energy distribution becomes non-Maxwellian [11]. For the electron velocity distribution function we use the two-term approximation [11] $f = f_0 + (\vec{v}/v)f_1$ (v is the electron velocity), which is valid for the majority of gases. If the frequency of variation of the plasma parameters is much less than the frequency ν of momentum transfer from electrons to neutral particles, then the system of equations for the isotropic and anisotropic parts of the electron velocity distribution function in electric and magnetic fields E and H has the form [8, 12]

$$\frac{\partial (nf_0)}{\partial t} + \frac{v}{3}\operatorname{div}(n\mathbf{f}_1) - \frac{en\mathbf{E}}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{f}_1) - S_0(nf_0) = 0,$$
$$\frac{v}{3}\nabla(nf_0) - \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} - \boldsymbol{\omega}\times\mathbf{f}_1 + \nu\mathbf{f}_1 = 0, \tag{2.1}$$

where $\nu = N\sigma_t v$; σ_t is the transport cross section of electron scattering on neutral particles; n is the electron density; $\omega = eH/mc$; S_0 is the collision integral averaged over angles. The electron distribution function is normalized by the condition

$$4\pi\int_0^\infty f_0 v^2 dv = 1.$$

If the characteristic size L of the nonuniformity and the time scale τ of variation of the electron energy distribution satisfy the conditions $\lambda_u \ll L$ and $\tau^{-1} \ll \nu_u$ (ν_u is the frequency of energy transfer from electrons to neutral particles), then the electron distribution function in the primary order f_{00} is determined by the solution of the system (2.1) without the terms describing the nonuniformity and nonsteadiness:

$$\frac{e^2}{3m^2v^2}\frac{\partial}{\partial v}\left\{\frac{v^2}{\omega^2+\nu^2}\frac{\partial f_{00}}{\partial v}\left[\nu E^2 + \frac{(\boldsymbol{\omega}\mathbf{E})^2}{\nu}\right]\right\} + S_0(f_{00}) = 0. \tag{2.2}$$

This function can be used as the zeroth approximation in the construction of a theory of disturbances in the parameters $(\tau\nu_u)^{-1}$ and λ_u/L.

We shall assume that the electron density and the electric field are nonuniform and nonsteady. Then, according to [10], the electron distribution function can be represented in the form

$$f_0(v) = f_{00}(v)\left[1 + \frac{a_j(v)}{n}\frac{\partial n}{\partial x_j} + \frac{b^{\perp}(v)}{E}\frac{\partial E_{\perp j}}{\partial x_j} + \frac{b^{\parallel}(v)}{E}\frac{\partial E_{\parallel j}}{\partial x_j} + \right.$$
$$\left. + \frac{c_j^{\perp}(v)}{E}\frac{\partial E_{\perp}}{\partial x_j} + \frac{c_j^{\parallel}(v)}{E}\frac{\partial E_{\parallel}}{\partial x_j} + \frac{d^{\perp}(v)}{E}\frac{\partial E_{\perp}}{\partial t} + \frac{d^{\parallel}(v)}{E}\frac{\partial E_{\parallel}}{\partial t}\right], \tag{2.3}$$

where $\mathbf{E}_{\parallel} = \boldsymbol{\omega}(\boldsymbol{\omega}\mathbf{E})/\omega^2$ and $\mathbf{E}_{\perp} = \mathbf{E} - \mathbf{E}_{\parallel}$. (Here and latter summation over the recurrent indices is understood.) Equations for the coefficients in the expansion (2.3) are presented in [10]. According to [10], the expression for the average directional velocity of the electrons has the form

$$w_i = \mu_{ij}E_j - \frac{D^*_{ij}}{n}\frac{\partial n}{\partial x_j} - \frac{D_i^{\perp}}{E}\frac{\partial E_{\perp j}}{\partial x_j} - \frac{D_i^{\parallel}}{E}\frac{\partial E_{\parallel j}}{\partial x_j} - \frac{D_{ij}^{\perp}}{E}\frac{\partial E_{\perp}}{\partial x_j} - \frac{D_{ij}^{\parallel}}{E}\frac{\partial E_{\parallel}}{\partial x_j} - \frac{D_i^{t\perp}}{E}\frac{\partial E_{\perp}}{\partial t} - \frac{D_i^{t\parallel}}{E}\frac{\partial E_{\parallel}}{\partial t},$$

where

$$\mu_{ij} = \frac{4\pi}{3}\frac{e}{m}\int_0^\infty \frac{v^3}{\omega^2+\nu^2}\frac{\partial f_{00}}{\partial v}\left(\nu\delta_{ij} + \varepsilon_{ikj}\omega_k + \frac{\omega_i\omega_j}{\nu}\right)dv,$$
$$D_{ij} = \frac{4\pi}{3}\int_0^\infty \frac{v^4 f_{00}}{\omega^2+\nu^2}\left(\nu\delta_{ij} + \varepsilon_{ikj}\omega_k + \frac{\omega_i\omega_j}{\nu}\right)dv,$$
$$D^*_{ij} = D_{ij} + J_i(f_{00}a_j), \quad D_i^{\perp,\parallel} = J_i(f_{00}b^{\perp,\parallel}),$$
$$D_{ij}^{\perp,\parallel} = E\frac{\partial D_{ij}}{\partial E_{\perp,\parallel}} + J_i(f_{00}c_j^{\perp,\parallel}), \quad D_i^{t\perp,\parallel} = J_i(f_{00}d^{\perp,\parallel}),$$

$$J_i(f) = -\frac{4\pi}{3}\frac{e}{m}\int\limits_0^\infty \frac{v^3}{\omega^2+v^2}\left(vE_i + \varepsilon_{inm}E_m\omega_n + E_m\frac{\omega_m\omega_i}{v}\right)dv,$$

and δ_{ij} and ε_{ijk} are Kronecker and Levi-Civita symbols. The tensor D^*_{ij} describes electron diffusion in the nonequilibrium plasma (the expression for D^*_{ij} was first obtained in [7]) and $D_i^{\perp,\parallel}$ and $D_{ij}^{\perp,\parallel}$ are the electron "thermodiffusion," since the quantity E determines the average electron energy. The tensor $D_i^{t\perp,\parallel}$ describes the nonclassical electron fluxes connected with the nonsteadiness of the electric field. According to [10], if ν does not depend on the electron velocity,

$$D^*_{ij} = D_{ij},\quad D_{ij}^{\perp,\parallel} = E\frac{\partial D_{ij}}{\partial E_{\perp,\parallel}},\quad D_i^{\perp,\parallel} = D_i^{t\perp,\parallel} = 0.$$

3. Electron Transport Coefficients in the Model Case

To determine the electron transport coefficients in the general case one must numerically solve a system of integrodifferential equations [10], but for certain models one can obtain an analytical solution of this problem.

Suppose the integral of collisions of electrons with atoms and molecules has the divergent form

$$S_0(f) = \frac{1}{2v^2}\frac{\partial}{\partial v}(v^3\nu_u f)$$

and the frequencies of elastic and inelastic collisions of electrons with neutral particles are power functions of the electron velocity: $\nu = \nu_0 v^p$, $\nu_u = \delta\nu$, $\delta = \delta_0 v^q$. As indicated in the introduction, this model reflects a number of concrete situations.

We shall assume that the magnetic field is perpendicular to the electric field and is directed along the z axis, i.e., $E_\parallel = 0$ and $E_\perp = E$. Then the solution of Eq. (2.2) has the form

$$f_{00} = C\exp\left[-\frac{3m^2\delta_0}{2e^2E^2}\left(\frac{\omega^2}{q+2}v^{q+2} + \frac{\nu_0^2}{2p+q+2}v^{2p+q+2}\right)\right]. \tag{3.1}$$

If the magnitude of the magnetic field is small and $\omega \ll \nu$, then the expression (3.1) is reduced to the well-known equation [9] without allowance for the magnetic field. In the other limiting case of $\omega \gg \nu$ Eq. (3.1) is simplified,*

$$f_{00} = C\exp[-(v/\alpha)^s],$$
$$\alpha^s = \frac{2s}{3\delta_0}\left(\frac{eE}{m\omega}\right)^2,\quad s = q+2,\quad C = \frac{s}{4\pi\alpha^3\Gamma\left(\frac{3}{s}\right)}. \tag{3.2}$$

It is just this case which will be considered below. Then the electron mobility and diffusion tensors are

$$\mu_{ij} = -\frac{p+3}{3}\frac{\Gamma\left(\frac{p+3}{s}\right)}{\Gamma\left(\frac{3}{s}\right)}\frac{e\nu(\alpha)}{m\omega^2}\delta_{ij} + \frac{e\omega_k}{m\omega^2}\varepsilon_{ijk},$$

$$D_{ij} = \frac{1}{3}\frac{\Gamma\left(\frac{p+5}{s}\right)}{\Gamma\left(\frac{3}{s}\right)}\frac{\alpha^2\nu(\alpha)}{\omega^2}\delta_{ij} - \frac{1}{3}\frac{\Gamma\left(\frac{5}{s}\right)}{\Gamma\left(\frac{3}{s}\right)}\frac{\alpha^2\omega_k}{\omega^2}\varepsilon_{ijk},$$

*The effective phase of the notation (3.2) is correct for the calculation of integral quantities of the type μ, D, etc. The distribution function itself may differ greatly from (3.2) owing to the second term in the exponent of (3.1).

where $\nu(\alpha) = \nu_0\alpha^p$ and $i, j = x, y$. We shall not write out the components of tensors with an index z, since the expressions for them do not depend on the magnetic field. With allowance for the renormalization, the expression for the diffusion tensor has the form

$$D_{ij}^* = D_{ij} + C_0 \frac{\alpha^2\nu(\alpha)}{\omega^2} e_i e_j, \quad e_i = E_i/E,$$

$$C_0 = \frac{p+3}{6} \frac{\Gamma\left(\frac{p+5}{s}\right)\Gamma\left(\frac{3}{s}\right) - \Gamma\left(\frac{p+3}{s}\right)\Gamma\left(\frac{5}{s}\right)}{\Gamma^2\left(\frac{3}{s}\right)} + S.$$

For $p > -3$ the quantity S can be written in the series form

$$S = \frac{(p+3)^2}{3s^2} \frac{\Gamma\left(\frac{p+3}{s}\right)}{\Gamma^2\left(\frac{3}{s}\right)} \sum_{k=1}^{\infty} \left[\frac{\Gamma\left(k-1+\frac{5}{s}\right)\Gamma\left(\frac{3}{s}\right) - \Gamma\left(k-1+\frac{5-p}{s}\right)\Gamma\left(\frac{p+3}{s}\right)}{\left(k-1+\frac{2-p}{s}\right)\Gamma\left(k+\frac{3}{s}\right)} - \right.$$

$$\left. - \frac{\Gamma\left(k+\frac{p+5}{s}\right)\Gamma\left(\frac{3}{s}\right) - \Gamma\left(k+\frac{5}{s}\right)\Gamma\left(\frac{p+3}{s}\right)}{\left(k+\frac{2}{s}\right)\Gamma\left(k+1+\frac{p+3}{s}\right)} \right].$$

The electron "thermodiffusion" tensors are written in the form

$$D_i^{\perp} = e_i \frac{\alpha^2\nu(\alpha)}{\omega^2} S,$$

$$D_{ij}^{\perp} = E\frac{\partial D_{ij}}{\partial E_{\perp}} + e_i \frac{\alpha^2\nu(\alpha)}{\omega^2}\left[\frac{2p}{s} S e_j + C_1 \varepsilon_{jmn} e_m \frac{\omega_n}{\nu(\alpha)} + C_2 e_j\right],$$

where

$$C_1 = \frac{2(p+3)}{s(2-p)} \frac{\Gamma\left(\frac{5-p}{s}\right)\Gamma\left(\frac{p+3}{s}\right) - \Gamma\left(\frac{5}{s}\right)\Gamma\left(\frac{3}{s}\right)}{\Gamma^2\left(\frac{3}{s}\right)},$$

$$C_2 = \frac{p+3}{3s(s+2)} \frac{(14+4p-3s)\,\Gamma\left(\frac{p+5}{s}\right)\Gamma\left(\frac{3}{s}\right) - (14-3s)\,\Gamma\left(\frac{p+3}{s}\right)\Gamma\left(\frac{5}{s}\right)}{\Gamma^2\left(\frac{3}{s}\right)}.$$

The expression for the tensor $D_i^{t\perp}$ describing the electron fluxes connected with the nonsteadiness of the electric field is reduced to

$$D_i^{t\perp} = C_1 \frac{m\alpha^2}{eE} e_i.$$

We note that as $\omega \to 0$ the equations obtained above coincide with the results of [9] with the substitution $p \to -p$. The quantity s is also different in this case. Here $s = q + 2$ while in [9] $s = 2p + q + 2$.

The results of a calculation of the dimensionless coefficients C_0, S, C_1, and C_2 as a function of p for s = 2 and 4 are presented in Table 1. It is seen that, as in the case without a magnetic field, the signs of these coefficients are determined by the sign of the derivative of the transport frequency of collisions of electrons with neutral particles with respect to the electron velocity. In the transition from one limiting case of $\omega \gg \nu$ to the other, however, the signs of S, C_0, C_1, C_2, and some of the electron transport coefficients change. This is connected with the fact that the dependence of the mobility of the electrons on their transport frequency changes in this case.

TABLE 1

p	$s=2$				$s=4$			
	C_0	S	C_1	C_2	C_0	S	C_1	C_2
−1	−0,26	−0,073	0,70	−0,75	−0,14	−0,054	0,10	−0,11
0	0	0	0	0	0	0	0	0
0,2	0,10	0,048	−0,16	0,28	0,032	0,016	−0,025	0,029
0,4	0,23	0,12	−0,32	0,63	0,067	0.036	−0,051	0.060
0,6	0,40	0,21	−0,50	1,06	0,11	0,058	−0,080	0,095
0,8	0.62	0,35	−0,69	1,60	0,15	0,083	−0,11	0,13
1	1,08	0,71	−0,91	2,26	0,20	0,12	−0,15	0,17
1,5	2,10	1,52	−1,57	4,67	0,34	0,22	−0,25	0.29
2	4,62	3,37	−2,54	8,75	0,57	0,40	−0,40	0,45

The experimental determination of the above-indicated electron transport coefficients will open up new possibilities for determining the cross sections for electron scattering on atoms and molecules and finding the regions of instability of a nonequilibrium weakly ionized plasma. The use of correct transport equations allows one to make sufficiently reliable calculations of the characteristics of a nonequilibrium weakly ionized plasma in electric and magnetic fields.

LITERATURE CITED

1. M. N. Vlasov, I. V. Kochetov, E. V. Mishin, V. G. Pevgov, and V. A. Telegin, "The electron energy distribution function and thermal balance of the ionospheric plasma in the presence of electric fields," Preprint No. 25 (338), Inst. Zeml. Magnet., Ionosf. Raspr. Radiovoln., Akad. Nauk SSSR, Moscow (1981).
2. L. A. Vulis, A. L. Genkin, and B. A. Fomenko, The Theory and Calculation of Magnetogasdynamic Flows in Channels [in Russian], Atomizdat, Moscow (1971).
3. T. S. Bondarenko and M. N. Turko, "Influence of a magnetic field on the kinetic characteristics of a plasma discharge," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, 8, No. 2 (1971).
4. Yu. V. Afonin, A. M. Orishich, and A. G. Ponomarenko, "Uniformity of a volume discharge controlled by an electron beam in a transverse magnetic field," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1979).
5. E. B. Wagner, F. J. Davis, and G. S. Hurst, "Time-of-flight investigations of electron transport in some atomic and molecular gases," J. Chem. Phys., 47, No. 9 (1967).
6. J. H. Parker and J. J. Lowke, "Theory of electron diffusion parallel to electric fields. I. Theory," Phys. Rev., 181, No. 1 (1969); H. R. Scullerud, "Longitudinal diffusion of electrons in electrostatic fields in gases," J. Phys. B, Ser. 2, 2, No. 4 (1969).
7. L. G. H. Huxley and R. W. Crompton, The Diffusion and Drift of Electrons in Gases, Wiley, New York (1974).
8. A. V. Timofeev, "Hydrodynamic transport equations for the weakly ionized plasma of a gas discharge," Zh. Tekh. Fiz., 40, No. 1 (1970).
9. N. L. Aleksandrov, A. P. Napartovich, and A. N. Starostin, "Transport equations in a nonequilibrium weakly ionized plasma," Fiz. Plazmy, 6, No. 5 (1980).
10. N. L. Aleksandrov, A. P. Napartovich, and A. N. Starostin, "Transport equations for electrons in a nonequilibrium weakly ionized plasma in electric and magnetic fields," Fiz. Plazmy, 9, No. 5 (1983).
11. V. L. Ginzburg and A. V. Gurevich, "Nonlinear phenomena in a plasma which is in a variable electromagnetic field," Usp. Fiz. Nauk, 70, No. 2 (1960).
12. I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, The Particle Kinetics of the Plasmas, Addison-Wesley, Reading, Mass. (1966).